**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:

The results for the Above Problem is drawn by Using python program Set2Q1.py

The probability that the service manager cannot meet his commitment is **0.266 (Option B)**

**Python Program:**

from scipy import stats

X= 1-stats.norm.cdf(60,loc=55,scale=8)

print("Required Probability is: ",X.round(4))

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

The results for the Above Problem is drawn by Using python program Set2Q2A.py & SetQ2B.py

1. **False.** Since Mean = 38, Standard Deviation = 6. By using this we can say most of the ages lies between (38+6) and (38-6) = 32 & 44.

Probability of Employees with age more than 44 is 0.1587

The Employees with age more than 44 will be approximately 0.1587×400 = 63

Probability of Employees with age in between 38 and 44 is 0.3413

The Employees with age more than 44 will be approximately 0.3413×400 = 137

Python Program:

# Z Score for 44

from scipy import stats

X= 1-stats.norm.cdf(44,loc=38,scale=6)

print("Probability of Employees age Older than 44 is: ",X.round(4))

# Z Score between 38 and 44

X1=1-stats.norm.cdf(44,loc=38,scale=6)

X2=1-stats.norm.cdf(38,loc=38,scale=6)

X3=X2-X1

print("Probability of Employees age in between 38 and 44 is:",X3.round(4))



Probability of Employees with under the age 30 is 0.91

The Employees with age under 30 will be approximately 0.91×400 = 364

Python Program:

# Z Score for 30

from scipy import stats

X= 1-stats.norm.cdf(30,loc=38,scale=6)

print("Probability of Employees age is 30 is: ",X.round(2))

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

From the Central Limit Theorem, any large sum of independent, identically distributed(iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean(µ) and the variance(),  and written as X~N(µ,)

Given *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent identically distributed random variables.

From the properties of normal random variables,

If X~N(,)  and  Y~N(,) are two independent identically distributed random variables then

The sum of normal random variables is given by:

X+Y ~N(+,  + ) ……………………………………………………………………..…….(1)

and the difference of normal random variables is given by:

X-Y ~N(-,  + ) ……………………………………………………………………..…..…….(2)

When  Z=aX, the product of X is given by

Z ~N(a,  )…………………………………………………………………………………..……..(3)

When  *Z=aX+bY*, the linear combination of X and Y is given by

Z ~N(a+ b,  + )………………………………………………………………...…(4)

From the above equations we can write 2 as

2~ N(2µ,) = 2~ N(2µ,) ……………………………………………………….…..(5)

+ ~ N(µ+µ, +) ~N(2µ,2) ………………………………………………………....(6)

From the above two equations we can say that, the mean is same but Variance of 2 is twice of +

2 – (+) ~ N(2µ-2µ, 4+) ~ N(0,6) ………………………………..……..(7)

The difference between 2and + says that the two given variables are identically and independently distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

The results were drawn by using python program Set2Q4.py

**Python Program:**

import scipy.stats as stats

# Z Score for 0.005

z1=stats.norm.ppf(0.005)

# Z score for 0.995

z2=stats.norm.ppf(0.995)

print('The Z Scores Z1 and Z2 are: ',z1.round(3),z2.round(3))

from the above python program we have Z score values as -2.576 & 2.576

We know that

Z= …………………………………………………………………….…….(1)

We can re-write the above equation as

= Z+…………………………………………………………..…….……..…(2)

From Given data we have Mean() = 100, Standard Deviation(σ) = 20 and

We got the Z scores values as -2.576 and 2.576

Substituting the above values in the equation we have

a = -2.576×20+100 = 48.5

b = 2.576×20+100 = 151.5

**The Required answer is Option D (48.5,151.5)**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

1. The results are drawn by using the python program Set2Q5A.py

**Python Program:**

import numpy as np

from scipy import stats

from scipy.stats import norm

Mean1 = 5

Mean2 = 7

Mean = Mean1+Mean2

print('The Mean Profit is Rs', Mean\*45,'Million')

STD1 = 3\*\*2

STD2 = 4\*\*2

STD = np.sqrt((9)+(16))

print('The Standard Deviation is Rs', STD\*45, 'Million')

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

* The Mean Profit is Rs **540** Million
* The Standard Deviation is Rs **225.0** Million
* Range is Rs (**99.00810347848784, 980.9918965215122**) in Millions

1. The Results are drawn by using python program Set2Q5B.py

**Python Program:**

import numpy as np

from scipy import stats

from scipy.stats import norm

Mean = 540

Std = 225

Z = stats.norm.ppf(0.05)

P = (Z \* Std) + Mean

print("The 5th percentile profit of the company is {} Millions".format(P.round(2)))

The 5th percentile profit of the company is **169.91** Millions (**170 Millions** Approximately)

1. The Results are drawn by using Python Program Set2Q5C.py

Python Program:

import numpy as np

from scipy import stats

from scipy.stats import norm

D1 = (stats.norm.cdf(0,5,3))

D2 = (stats.norm.cdf(0,7,4))

print("The Probability of Division-1 making Loss is: ",D1.round(4)\*100)

print("The Probability of Division-2 making Loss is: ",D2.round(4)\*100)

The Probability of Division-1 making Loss is: 4.78

The Probability of Division-2 making Loss is: 4.01

Therefore Division-1 Will have More probability for making loss than Division-2